

A Survey on Recent Off-Shore Wind Farm Layout Optimization Methods

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Abstract. Energy demands are increasing in a global scale. Higher demand results in higher energy costs. This is while climate change and other environmental concerns as well as lack of energy resources pushes the energy sector towards employing renewable and green energy sources. This is why, the importance of wind farms is on a rise and many countries/companies consider wider utilization of this renewable energy resource. Technically speaking, the main challenge in this area is to reduce the cost of energy production and to maximize the energy outputs of the farms. Given such a goal, the layout design of the wind farm, say, the formation of wind turbines, is a deciding factor in increasing the performance of the farms as well as reducing maintenance costs. That is why, many researchers have directed their research towards wind farm layout optimization problem. In this study, a brief survey of recent researches on the subject is given.

1 Introduction

The ever increasing demand for energy and environmental concerns has resulted in higher investments on green and renewable energy sources. Wind energy, converted to electricity by wind turbines is one of the areas which has received a lot of attention in recent years. There are two types of wind farms: *on-shore* and *off-shore*. On-shore wind farms attract quite a lot of complains from various organizations and communities with an interest in preserving local landscapes. In addition, there are claims that the wind quality (velocity, continuity and etc.) are higher in off-shore areas, which is yet another reason in an increase of investment and attention in off-shore wind farms. However, regardless of the type of the farm, the main challenge in this area is to maximize the performance of the farm while reducing the costs such as maintenance and cabling. Wind farm layout design is a determining factor which affects both the cost and the performance of the farm. Due to this reason, many scientists have directed their research towards optimizing the wind farm layout. The particular problem which is the focus of this survey is Off-Shore Wind Farm Layout Optimization (OWFLO) problem. However, on-shore studies are also covered here as long as studies related to on-shore layout optimization do not deal with constraints and assumptions specific

to the on-shore aspect of the problem. Interested reader may also refer to [1] where an extensive survey of the approaches proposed to solve the wind farm layout optimization problem is provided.

This study provides a survey on some of the recent researches on the OWFLO problem. However, prior to describing the proposed methods in more detail, there is a need to deal with basic terminologies. This is done in Section.2. Following the introduction of the elements of the OWFLO problem various approaches which were proposed to solve the OWFLO problem are discussed in Section.3. In Section.4, a conclusive discussion is laid out and a roadmap for future work is proposed.

2 Terminology

The layout of a wind farm determines the placement of the turbines with respect to each other as well as their location with respect to the electrical connection point on shore. Thus, the spacing between the turbines and their distance from the shore are two deciding factors when estimating the performance of the farm.

2.1 Trade-offs

There are few trade-offs that need to be considered while designing a wind farm layout. For example, the farther the farm is from the shore the cost of cabling and maintenance rises. Also, as the distance from the shore increases, usually the depth of the water increases as well which yields an increase in the cost of support structures required for the turbines. On the other hand, a larger distance from the shore typically results in higher wind speed and subsequently higher energy output.

The spacing between the turbines is another parameter which introduces another layer of trade-offs. A larger space between the turbines increases the maintenance and inter-turbine cabling costs. This is while, higher distance between turbines increases the performance of each turbine. This is due to the *wake loss* caused by each turbine. Each turbine absorbs some of the wind energy and thus the wind in the wake of a turbine is less effective with higher turbulence in the downstream direction of the blades. This loss of wind energy in the wake of a turbine is referred to as the wake loss. It is important to understand that not only wake effect causes a reduction in energy production of subsequent (downstream) turbines, it also implicates a fluctuation in the energy output of the wind farm as a whole. This fluctuation reduces the predictability of the performance of the wind farm which in turn decreases the reliability of the farm.

2.2 The Weibull Model

One of the first things that has to be considered when investigating the OWFLO problem is the probability distribution of the wind. In the literature, the probability density function of the wind at the site is dominantly modelled using a

Weibull distribution [2]. This assumption is open to counter-arguments as it may not hold for all the sites.

The Weibull model has two parameters: the scale factor c and the shape factor k . The distribution equation as a function of the wind speed U is given in Eq.1

$$p_U(U) = (k/c)(U/c)^{k-1} \exp(-(U/c)^k) \quad (1)$$

Subsequently, the energy output of a wind turbine can be computed according to the following equation (Eq.2).

$$P(\bar{w}) = \int_0^{360} p_\theta(\theta) \left[\int_0^\infty P_W(U) \left(\frac{k(\theta)}{c_i(\theta, w)} \right) \left(\frac{U}{c_i(\theta, w)} \right)^{k(\theta)-1} \exp\left[-\left(\frac{U}{c_i(\theta, w)} \right)^{k(\theta)}\right] dU \right] d\theta \quad (2)$$

In Eq.2, θ is the direction of the wind, w turbine's distance from the shore while $P(\bar{w})$ and $P_W(U)$ are the average power output and the power curve of the turbine respectively. Please note that, the power curve of a turbine ($P_W(U)$) is given by the turbine manufacturer and usually has the form of a discrete dataset. Fitting a continuous function to such data is recommended [3].

It is clear from the Eq.2 that, the Weibull parameters (c and k) are functions of the wind directions and shore distance. However, wind does not always blow from a fixed direction. This describes the first integral in Eq.2 and therefore we can say that the rest of the equation is weighted by the probability of the wind blowing from a given direction.

2.3 The Wake Loss Models

The wake loss was literally defined in this section. Several analytical wake models have been proposed so far. Here, we investigate those wake models on which the literature is heavily relied on.

Jensen wake model The Jensen model first proposed in [4] assumes a linearly expanding wake behind a given turbine. This model is only suitable when the spacing between turbines is more than 3 or 4 time the rotor diameter. A snapshot of the Jensen model is given in Figure.1.

In the wake of a turbine, the wind loses a fraction of it's speed. This reduction is referred to as the velocity deficit and is formulated by Jensen wake model in the following equation (Eq.3).

$$Vel_{Def} = 1 - \frac{V}{U} = 1 - \frac{\sqrt{1 - C_T}}{(1 + 2\kappa X/D)^2} \quad (3)$$

In Equation.3, U is the free stream wind speed, V is the wind speed in the wake of the turbine, C_T is the thrust coefficient of the turbine, κ is the wake spreading constant, X is the distance behind the upstream turbine and D is the turbine rotor diameter. These variables are further marked in Figure.1. The

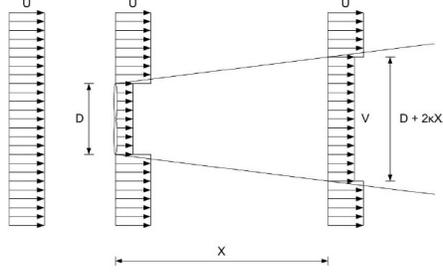


Fig. 1. The Jensen wake model description (adopted from [4]).

velocity deficit in Equation.3 is for the cases when a turbine suffers from a single turbine wake. For cases where there are many other turbines (i.e. N turbines) the total wake loss is computed according to Equation.4.

$$Vel_{Def_{Total}} = \sqrt{\sum_{i=1}^N Vel_{Def_i}^2} \quad (4)$$

Please note that $Vel_{Def_{Total}} = 0$ when there are no upstream turbines and $Vel_{Def_{Total}} > 0$ otherwise.

Park wake model This model (the modified version of it) was first proposed in [5]. In this model, briefly, the wake effects on a turbine i change the wind resource available to it along different directions by reducing the scale parameter of the Weibull distribution estimated for the entire farm. The velocity deficit is thus calculated as in the following equation.

$$Vel_{Def(i,j)} = u(\delta_{i,j} - \alpha) \left(\frac{1 - \sqrt{1 - C_T}}{(1 + \kappa X/D)^2} \right) \quad (5)$$

where $X = d_{i,j}$ is the distance between turbines i and j , u is the unit step function, C_T is the thrust coefficient of the turbine, κ is the wake spreading constant and $\alpha = \tan^{-1}(\kappa)$. Please note that $d_{i,j} = ||o||$ where $o = (x_i - x_j) \cos \theta + (y_i - y_j) \sin \theta$. In Eq.5, $\delta_{i,j}$ is calculated as in the following equation.

$$\delta_{i,j} = \cos^{-1} \left\{ \frac{o + R/\kappa}{\sqrt{((x_i - x_j) + (R/\kappa) \cos \theta)^2 + ((y_i - y_j) + (R/\kappa) \sin \theta)^2}} \right\} \quad (6)$$

Again, the total velocity deficit is given by the Equation.4.

2.4 Frandsen wake model

This model which determines the growth of individual wakes was first proposed in [6]. The Frandsen wake model employs the control volume concept that relates

the thrust and power coefficients to the velocity deficit. The growth of the wake front behind any turbine j is given by the following equation.

$$D_{wake,j} = (1 + 2\alpha\bar{s})D_j \quad \text{where } \bar{s} = s/D_j \quad (7)$$

where $D_{wake,j}$ is the diameter of the expanding wake front at a distance s behind Turbine j . The parameter α is the wake spreading constant [6], which is determined using the formula

$$\alpha = \frac{0.5}{\ln\left(\frac{z_H}{z_0}\right)} \quad (8)$$

where z_H and z_0 are the average hub height of the turbines and the average surface roughness of the wind farm region, respectively. The wind velocity in the wake is given by:

$$U = \left(1 - \frac{2a}{(1 + 2\alpha s)^2}\right)U_j \quad (9)$$

where a is the induction factor, which can be determined from the coefficient of thrust (C_t). The latter is one of the design characteristics of a turbine rotor. Eq.9 is equivalent to the expression suggested in the Park wake model.

2.5 Typical objective functions

Though most of the literature tends to propose a different cost model which serves their local concerns best, in this section it has been tried to provide the reader with few well defined cost models available in the literature.

Annual Energy Product (AEP) The expected AEP is defined as in Eq.10 which takes into consideration the wake effects and allows quantifying the benefits in terms of profit from the location strategy. As the wake effect depends on the wind distribution function, and since the Weibull distribution is dominantly considered for this matter, the AEP is usually measured in terms of the Weibull distribution.

$$AEP = \frac{k(\theta)}{c(\theta)} \left(\frac{U}{c(\theta)}\right)^{k(\theta)-1} e^{-\left(\frac{U}{c(\theta)}\right)^{k(\theta)}} \quad (10)$$

where k and c are shape and scale factors of the Weibull distribution, U is the wind speed at the hub height and θ is the wind direction.

Levelized cost of energy (LCOE) Described in details in [3], LCOE is common means of evaluating energy production costs. LCOE is best described by the following equation.

$$LCOE = (C_C.FCR + C_{O\&M})/AEP \quad (11)$$

Where, C_C is the total construction cost of the wind farm (turbine, support structure, cabling and etc.), FCR is the fixed charge rate, a present value factor that includes debt and equity costs, taxes, and insurance, $C_{O\&M}$ is the annual operation and maintenance costs and AEP is the Annual Energy Production of the wind farm Eq.10 (kWh per annum). The $LCOE$ itself is measured in dollars per kWh.

Levelized production cost (LPC) LPC, best described in [7], is formulated using the following equation.

$$LPC = \frac{C_{iw}}{a \cdot E_a} + \frac{C_{O\&M}}{E_a} \quad (12)$$

where C_{iw} is the total investment cost, $C_{O\&M}$ is the annual operation and maintenance costs, a is the annuity factor and E_a is the annual energy production of the farm measured in kWh. The LPC is measured by dollars per kWh.

Mosetti's cost model Mosetti, one of the pioneers in wind farm layout optimization, proposed a cost model formulated as in the following equation.

$$cost = N \left(\frac{2}{3} + \frac{1}{3} e^{-0.00174N^2} \right) \quad (13)$$

where N is the number of turbines.

3 Approaches

In [7], the Jensen model is used and few algorithms such as Genetic Algorithm (GA), Gradient Search Algorithm (GSA), Greedy Heuristic Algorithm (GHA), Simulated Annealing (SA) and Pattern Search Algorithm (PSA) were experimented to tackle the OWFLO problem. The problem representation is binary. That is, the farm layout is considered to be a grid and depending on whether a turbine is placed in each grid cell or not the value corresponding to the grid cell can be 1 or 0 respectively. Generally, the GA framework and the assumptions are similar to that of [8] which is the first article which dealt with OWFLO problem. It can be observed in the paper that GA and the combination of GA-GHA, where the GHA (also a population based method) improves upon the solution provided by GA outperform other algorithms. The same methods, yielding similar comparative results, are also applied to a real-world wind farm in Massachusetts, USA. Please note that parameters included in the fitness function are: annual maintenance cost, total investment cost and the annual energy output.

In [9] a method based on the Monte-Carlo Simulation (MCS) is proposed. The authors employed the Jensen wake model and the problem representation is of a grid style, where the number of turbines is not fixed. The cost is estimated using Eq.13. The proposed method is then compared to two GA approaches (one of which is the Mosetti's approach [8]) and tested all the methods on a single test

case (constant and unidirectional wind speed). The results show that the MCS approach provides a better solution. Although the best solution found by MCS method utilizes more turbines (in fact it has 3 – 4 more turbines than the best solution offered by both GA algorithms), the authors argue (very reasonably) that since the fitness is the cost/power output ratio, the lower fitness generated by their method is sufficient to say that it has outperformed the two GA based methods. This claim seems to be very convincing as it is supported by the results.

MCS has also been employed in a real world problem in [10]. The real world problem is a wind farm with a capacity of four turbines in Minnesota State University, USA. The authors has chosen to use the Jensen wake model, similar to the study above.

In [11] a bi-objective Evolutionary Strategy (ES) called SPEA in the literature is proposed. Although the proposed method is for on-shore sites, it is one of the most frequently cited approaches in the literature which assumes a roughly flat terrain similar to off-shore sites. The problem is represented as the (x, y) coordinates of the turbines within the farm with a spacing constraint. The algorithm uses Jensen’s wake model and assumes a continuous Weibull distribution of the wind. It also assumes that the number of turbines is given beforehand. SPEA also proposes a formulation by which it is determined if a specific turbine is within the wake effect of other turbine(s). The objectives of the SPEA algorithm are to maximize the energy output while minimizing the terrain constraint violation. Two wind scenarios are considered in this paper, one of which is actually according to the collected data from a real-world industrial farm. The proposed method has been successfully applied to both scenarios with ”acceptable” results.

The same wind scenarios are also the subject of study in [12] were Covariance Matrix Adaptation Evolutionary Strategy (CMA-ES) is compared to the SPEA method proposed in [11]. However, the results are not significantly different. The wake model used in this study is the Park model. The reason for such a choice is not clear. The authors also confirm in their paper, that the deep array model (in other words the Jensen model) is more suitable for large wind farms than the model used in their work.

In [13] an Evolutionary Algorithm (EA) seeded by a greedy heuristic algorithm which functions as a construction heuristic is proposed for the OWFLO problem. The wake model in use is Jensen’s wake model. The problem representation is the set of (x, y) coordinates of each turbine’s candidate location within the farm. The main contribution of the paper though is the inclusion of the flexible wind farm shapes, an orography model (variations on wind speed which is somehow exclusive to on-shore sites due to hills) and a cost model based on the benefit/investment terms. This cost model is given in Eq.14.

$$cost = B - t - N.C_i - \sum_{i=1}^N \sum_{i < j} C_{ij}^C \quad (14)$$

In Eq.14, C_i is the installation cost of turbine i , C_{ij}^C is the connection costs between turbines i and j , b_t is the net benefit obtained from the energy produced in t years and N is the total number of turbines.

In [14] a viral based optimization algorithm is proposed to solve the wind farm micro siting problem. Though the study is not dedicated to off-shore sites, it well can be used as the assumptions are not restricted to those of on-shore wind farms. The cost model of the wind farm used in this paper is formulated as in Eq.13.

In [15] heuristic methods which address both on-shore and offshore wind farm layout designs are proposed. Two scenarios are investigated. In the first scenario the wind farm region as well as the number of turbines are fixed and given while in the second scenario finite locations in the region are feasible and the number of turbines is no longer a constant fixed value. A new wake model is provided which is based on Jensen's wake model, however it is slightly different as it assumes a Gaussian distribution across the wake. The proposed heuristics are modifications to Nelder-Mead algorithm as well as other heuristics (refer to [15] for further detail).

In [16] a wind farm micro and macro-siting toolkit based on genetic algorithm is proposed. The toolkit is developed during a wind farm siting project at Rhode Island, USA. The toolkit estimates the wind probability distribution model with a Weibull function and the wake model employed is the Jensen's model while the inclusion of more sophisticated wake models is marked as the future work. However, the most striking aspect of the paper is the cost model in which various cost parameters such as social, ecological, cultural and environmental costs are measured and included in the toolkit.

In [17] the Unrestricted Wind Farm Layout Optimization (UWFLO) method has been proposed. In this method, some typical restrictions imposed on the location of the turbines within the wind farm layout design are ignored. This in part due to the fact that the turbine rotor diameter is considered to be variable, giving the algorithm the power to generate a more flexible wind farm layout which is less affected by the wake loss. Please note that, it is the wake loss which restricts the distance of subsequent turbines to be 3 or 4 times the rotor diameter. Thus, having the option to choose from various ranges of rotor diameters relaxes this constraint. The authors have used Constrained Particle Swarm Optimization (CPSO) for optimization purposes and the wake model employed is the Frandsen wake model. Also, the data, against which the proposed approach has been tested, is acquired from wind tunnel experiments on a scaled down wind farm which is particularly interesting. The cost model employed in this study is based on quadratic response surfaces (for more detail refer to [17]) which is very much different than that of conventional cost models used in the literature. The proposed cost model explicitly considers the effect of turbine rotor diameter on the cost.

In [18] a multi-objective on-shore wind farm layout optimization method based on NSGA-II is proposed which considers an additional constraint, namely noise propagation of the wind farm. Similar to many other approaches, the au-

thors have employed Jensen’s wake model in their study and the problem has been represented as a rectangular grid. The cost model considered in this study is only the Annual Energy Production (AEP Eq.10) as the authors try to maximize the energy production without considering other economic factors.

In [19] a new wake model derived from a Computational Fluid Dynamics (CFD) simulation of the wind turbine is given. A Genetic Algorithm (GA) framework has been used to test the CFD model as well as the Jensen model. The two models are then compared against each other for various scenarios which differ in their consideration of the wind direction (cases in which uni-directional and four directional wind are considered). The authors claim that the CFD model takes into account some several physical issues (real atmospheric profiles, the presence of the ground, an appropriate turbulence model) which were ignored in previous models (i.e. Jensen’s model). However, the experimental results indicates that the Jensen model provides better solutions (less cost, more energy output). It is not clear though if the inferior performance of the CFD model is due to it’s realistic approach towards formulating the problem or it is simply the case that the Jensen model is more accurate. The cost model used in this study is as in Eq.13.

[20] approaches the OWFLO problem from a mathematical programming perspective. Their approach has two steps. First, a construction heuristic is employed to generate an initial solution. In the second step, this initial solution is then fed to a non-linear mathematical programming method which uses this initial solution. The authors employ the Jensen method to model the wake effect and the problem representation is the (x, y) coordinates of the turbines. The proposed approach has been tested on a wind farm in northern Germany and marked a 3.52% increase in the performance of the wind farm. The cost model used in this study is the expected AEP (Eq.10).

4 Conclusion

A brief and somehow shallow survey of the recent methods in wind farm layout optimization is given in this work. The various cost models, wake effect modelling methods as well as recently proposed approaches are covered. Studying the proposed approaches leaves us with the following conclusions:

- Cost models: the proposed approaches are very sensitive to the cost models and no general and comprehensive cost model has been proposed. However, it is important to design a good and reliable cost model before diving into any project.
- The lack of benchmark cases: Looking at the literature, the usual case is that the authors try to evaluate their proposed approaches on real-world problems. Obviously this has it’s own advantages. However, the lack of benchmarking cases makes comparison between approaches difficult.
- Wake models: several wake models have been proposed and tested in various proposals. However, it is not clear which wake model describes the wake of the turbines best.

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