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A Tensor-based Approach to Nurse Rostering

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1 Introduction

Hyper-heuristics are high level improvement search methodologies exploring space of heuristics [4]. According to [5], hyper-heuristics can be categorized in many ways. A hyper-heuristic either selects from a set of available low level heuristics or generates new heuristics from components of existing low level heuristics to solve a problem, leading to a distinction between *selection* and *generation* hyper-heuristics, respectively. Also, depending on the availability of feedback from the search process, hyper-heuristics can be categorized as *learning* and *no-learning*. Learning hyper-heuristics can further be categorized into online and offline methodologies depending on the nature of the feedback. Online hyper-heuristics learn *while* solving a problem whereas offline hyper-heuristics process collected data gathered from training instances prior to solving the problem.

Nurse rostering is a highly constrained scheduling problem which was proven to be NP-hard (Karp, 1972) in its simplified form. Solving a nurse rostering problem requires assignment of shifts to a set of nurses so that 1) the minimum staff requirements are fulfilled and 2) the nurses' contracts are respected [3]. The problem can be represented as a constraint optimisation problem using 5-tuples: (i) set of nurses, (ii) set of days (periods) including the relevant bits from the previous and upcoming schedule, (iii) set of shift types, (iv) set of skill types and (v) constraints.

In this study, a novel selection hyper-heuristic approach is employed to tackle the nurse rostering problem. The proposed framework is a single point based search algorithm which fits best in the online learning selection hyper-

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heuristic category, even if it is slightly different than the other online learning selection hyper-heuristics. A selection hyper-heuristic has two main components: heuristic selection and move acceptance method. While the task of the heuristic selection is to select low level heuristics based on a strategy, the acceptance method decides whether or not the solution produced by the selected heuristic shall be accepted. Over the years many heuristic selection and move acceptance methods have been proposed. Examples of heuristics selection strategies are Simple Random (SR) and Random Gradient (RG)[7], Choice Function (CF) [7], Reinforcement Learning (RL) [11] and Tabu Search (TS) [6]. Improvement Only (IO), Improvement Equal (IE), Naive Acceptance (NA), Simulated Annealing(SA) and Late Acceptance (LA) are few examples of move acceptance mechanisms.

In our proposed approach, the trace of a selection hyper-heuristic combining random heuristic selection with a Naive Acceptance (NA) method is represented as a 3-rd order tensor. Factorization of this tensor results in basic factors, which are then analysed and used to partition the low level heuristics into two halves. The first half of the low level heuristics are considered to perform well with NA, while the remaining half are associated with the Improvement Equal (IE) acceptance method. The proposed hyper-heuristic approach then periodically switches between the two move acceptance methods mixing only the relevant low level heuristics associated with them.

2 A Novel Hyper-heuristic Approach: Tensors for Analysing the Space of Heuristics

Tensors are multidimensional arrays. The order of a tensor is the number of dimensions it covers. Tensor decomposition (a.k.a tensor factorization) reduces the dimensionality of the original tensor while keeping the multi-dimensional nature of the data. This helps with preserving the correlation between various modes of data. As well as being somewhat immune to noise, the tensor factors are useful for generalization purposes. The literature is rich of a range of tensor factorization methods such as Higher Order SVD (HOSVD) [10] and Parallel Factor (a.k.a PARAFAC or CANDECOMP or CP) [9]. These methods are often generalization of the Singular Value Decomposition (SVD) method to higher dimensions.

In this study, the search history of a simple random hyper-heuristic with naive acceptance is represented as a 3-rd order tensor. The first two dimensions of this tensor correspond to the current and previous heuristic indexes, called by the underlying hyper-heuristic, while the iteration is projected to the 3rd dimension. The goal is to hybridise multiple hyper-heuristics (SR-IE and SR-NA) and use the most suitable one depending on the randomly chosen low level heuristic at each iterative step under a selection hyper-heuristic framework. This process requires partitioning of heuristics and assigning each group to the best hyper-heuristic. Using the CP decomposition method, the tensor is reduced to a basic factor (frame), interpreting which reveals the pairs of heuristics performing well together under naive acceptance. Analysing these basic factors, scores are associated to each low level heuristic where half of the heuristics which rank highest form a group of heuristics performing well under Naive Acceptance (NA) mechanism. The remaining half form a secondary group which are then applied together with the Improvement Equal (IE) acceptance mechanism. This online learning process is employed for a fixed time in each run on a given problem instance. Then the second stage starts operating. The novel hyper-heuristic approach periodically switches between the two hyper-heuristics in fixed periods and the low level heuristic sets associated with them. Thus, a Tensor-based Hyper-heuristic (THY) emerges.

3 Empirical Results

Hyper-heuristics Flexible Framework (HyFlex) is a Java based software library for the implementation and comparison of various hyper/meta-heuristics across different problem domains [12]. One of those problem domains is the nurse rostering (Personnel Scheduling) implementing 12 low level heuristics of various types: 1 mutation, 5 hill climbing, 3 ruin and re-create and 3 crossover heuristics. Some previously proposed hyper-heuristics do not discriminate the nature of low level heuristics while some others do take that into account in their design. THY is of former type. Since the crossover heuristics are binary operators requiring additional maintenance (for the second argument), for simplicity, they are discarded by THY. Using the HyFlex v1.0 Java library [1], THY is implemented for nurse rostering and tested on 8 benchmark instances [2]. The best and average cost obtained by THY for each instance are presented in Table 1 in comparison to the previously proposed approaches of scatter search (SS) [2] and variable depth search (VDS) [1]. The tensor-based approach to nurse rostering is generating high quality solutions in a reasonable amount of time.

Table 1 The comparison of solutions obtained by previously proposed approaches [8] to the ones found by THY across 31 runs for some nurse rostering benchmark problem instances. BKN denotes the best known solution.

		Our Approach			SS [2]	VDS [1]
Problem	BKN	Best	Avr.	Time (s)	Best	Best
BCV-3.46.1	3280	3284	3312.23	354	3351	-
BCV-A.12.1	1294	1384	1643.90	435	1600	-
BCV-A.12.2	1875	1940	2152.03	277	2180	-
Ikegami 3d1	2	13	19.45	258	-	13
Ikegami 3d1.1	3	15	21.32	409	-	14
Ikegami 3d1.2	3	16	21.74	318	-	9
ORTEC01	270	300	338.74	390	-	465
ORTEC02	270	300	325.97	258	-	510

[1] http://hyflex.org/

[2] http://cs.nott.ac.uk/~tec/NRP/

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